RELATIONSHIPS WITHIN TRIANGLES

apter 5

Geometry 5





Length should be ½ They should be parallel





UV, WV

UW

UW = ½ ST VT = ½ ST UW = VT = 81

5.1 Midsegment Theorem and

Coordinate Proof

- Coordinate Proof
- 1. Use the origin as a vertex or center.
- 2. Place at least one side of the polygon on an axis.
- 3. Usually keep the figure within the first quadrant.
- Use coordinates that make computations as simple as possible.
- You will prove things by calculating things like slope, distance, and midpoints







Since we're finding midpoints, it is convenient to use 2a, 2b, and 2c so that the midpoints are whole numbers.

Midpoint calculations:

midpoint = $((x_1 + x_2)/2, (y_1 + y_2)/2)$ ((0 + 2a)/2, (0 + 0)/2) = (a, 0)((2a + 2b)/2, (0 + 2c)/2) = (a + b, c)

Parallel calculations:

slope = $(y_2 - y_1)/(x_2 - x_1)$ m = (2c - 0)/(2b - 0) = c/bm = (c - 0)/(a+b - a) = c/bslopes are the same so lines are parallel

Length calculation:

dist = $V((x_2 - x_1)^2 + (y_2 - y_1)^2)$ $V((2b - 0)^2 + (2c - 0)^2) = V(4b^2 + 4c^2) = V(4(b^2 + c^2)) = 2V(b^2 + c^2)$ $V((a+b-a)^2 + (c-0)^2) = V(b^2 + c^2)$

5.1 Midsegment Theorem and

Coordinate Proof

 298 #2-18 even, 24-32 even, 36, 40, 42, 44, 48-52 even = 21 total

5.1 Answers and Quiz

□ <u>5.1 Answers</u>

□ <u>5.1 Quiz</u>





Since JK is \perp bisector, then NK = LK (\perp bisector theorem). 6x - 5 = 4x + 1 \rightarrow 2x - 5 = 1 \rightarrow 2x = 6 \rightarrow x = 3 Find NK: 6x - 5 \rightarrow 6(3) - 5 = 13

Since MN = ML, M is equidistant from each end of NL. Thus by then Converse of the Perpendicular Bisector Theorem, M is on the perpendicular bisector.

- Find the perpendicular bisectors of a triangle
- Cut out a triangle
- Fold each vertex to each other vertex
 - The three folds are the perpendicular bisectors
- What do you notice?
 - Perpendicular bisectors meet at one point
- Measure the distance from the meeting point to each vertex
- What do you notice?
 - The distances are equal



Hot pretzels are sold from store at A, B, and E. Where could the pretzel distributor be located if it is equidistant from those three points?



- Circumcenter
 - The point of concurrency of the perpendicular bisectors of a triangle.
 - If a circle was circumscribed around a triangle, the circumcenter would also be the center of the circle.



- 306 #2-16 even, 20, 22, 26, 28, 30, 34-40 even
 = 17 total
- □ Extra Credit 309 #2, 4 = +2

5.2 Answers and Quiz

□ <u>5.2 Answers</u>

□ <u>5.2 Quiz</u>





 $3x + 5 = 4x - 6 \rightarrow 5 = x - 6 \rightarrow x = 11$

 $5x = 6x - 5 \rightarrow -x = -5 \rightarrow x = 5$

No, you need to know that SP \perp QP and SR \perp QR

5.3 Use Angle Bisectors of Triangles





Find NF by using the Pythagorean theorem. $16^2 + NF^2 = 20^2 \rightarrow 256 + NF^2 = 400 \rightarrow NF^2 = 144 \rightarrow NF = 12$ Since N is the incenter, NF = EN = 12

5.3 Answers and Quiz

□ <u>5.3 Answers</u>

□ <u>5.3 Quiz</u>

5.4 Use Medians and Altitudes



5.4 Use Medians and Altitudes

- Each path goes from the midpoint of one edge to the opposite corner. The paths meet at P.
- \square If SC = 2100 ft, find PS and PC.

 \square If BT = 1000 ft, find TC and BC.



 \square If PT = 800 ft, find PA and TA.

 $PC = 2/3 \text{ SC} \rightarrow PC = 2/3 \text{ (2100)} = 1400 \text{ ft} \rightarrow PS = 700 \text{ ft}$

T is midpoint of BC. TC = 1000 ft, BC = 2000 ft

PT = 1/3 TA \rightarrow 800 = 1/3 TA \rightarrow 2400 ft = TA, PA = 1600 ft



There is nothing terribly interesting about the orthocenter. In an acute triangle, the orthocenter is inside the triangle. In a right triangle, the orthocenter is on the triangle at the right angle. In an obtuse triangle, the orthocenter is outside of the triangle.

5.4 Use Medians and Altitudes

In an isosceles triangle, the perpendicular bisector, angle bisector, median, and altitude from the vertex angle are all the same segment.





Draw the other two altitudes (from A and C). They will be outside the triangle.

OQ is also the perpendicular bisector, angle bisector, and median. P (-h, 0)

5.4 Use Medians and Altitudes

 $\Box \ \Delta PQR \text{ is isosceles and segment } \overline{OQ} \text{ is an altitude. What else } do you know about } \overline{OQ}? What are the coordinates of P?$



Draw the other two altitudes (from A and C). They will be outside the triangle.

OQ is also the perpendicular bisector, angle bisector, and median. P (-h, 0)



ΔABC is isosceles, BD is a me	dian (given)	
$BA\congBC$	(def. Isosceles)	
$AD\congDC$	(def. Median)	
$BD\congBD$	(reflexive)	
$\Delta ABD \cong \Delta CBD$	(SSS)	
$\angle ABD \cong \angle CBD$	$(def \cong \Delta)$ (CPCTC)	
BD is an angle bisector	(def angle bised	ctor)

5.4 Use Medians and Altitudes

- 322 #2-26 even, 34, 40, 42, 46-54 even = 20 total
- □ Extra Credit 325 #2, 4 = +2

5.4 Answers and Quiz

□ <u>5.4 Answers</u>

□ <u>5.4 Quiz</u>



Smallest angle Largest side

5.5 Use Inequalities in a Triangle

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

121

30°

29°

R-

List the sides in order from shortest to longest.

ST, RS, RT



Can't be done, short side don't touch Can't be done, forms a line Can be done, isosceles triangle

5.5 Use Inequalities in a Triangle

A triangle has one side of 11 inches and another of 15 inches. Describe the possible lengths of the third side.

 $11 + x > 15 \rightarrow x > 4$ $15 + x > 11 \rightarrow x > -4 \text{ (not useful)}$ $11 + 15 > x \rightarrow 26 > x$ Combine 1st and 3rd: 4 < x < 26

Short cut: subtract to get smallest, add to get largest



 $x + 11 + 2x + 10 > 5x - 9 \rightarrow 3x + 21 > 5x - 9 \rightarrow 30 > 2x \rightarrow 15 > x$

 $2x + 10 + 5x - 9 > x + 11 \rightarrow 7x + 1 > x + 11 \rightarrow 6x > 10 \rightarrow x > 10/6 \rightarrow x > 1 2/3$

 $x + 11 + 5x - 9 > 2x + 10 \rightarrow 6x + 2 > 2x + 10 \rightarrow 4x > 8 \rightarrow x > 2$

Choose narrowest interval: 2 < x < 15

5.5 Answers and Quiz

□ <u>5.5 Answers</u>

□ <u>5.5 Quiz</u>

5.6 Inequalities in Two Triangles and Indirect Proof See Mr. Wright's demonstration with the metersticks.

How does the third side compare when there is a small angle to a big angle?

Use two meter sticks to demonstrate the SAS Inequality and SSS Inequality Have two meter sticks form two sides of the Δ and have the kids imagine the third side.

Hinge Theorem

If 2 sides of one Δ are congruent to 2 sides of another Δ , and the included angle of the 1st Δ is larger than the included angle of the 2nd Δ , then the 3rd side of the 1st Δ is longer than the 3rd side of the 2nd Δ .



15 60

Converse of the Hinge Theorem

If 2 sides of one Δ are congruent to 2 sides of another Δ , and the 3rd side of the first is longer than the 3rd side of the 2nd Δ , then the included angle of the 1st Δ is larger than the included angle of the 2nd Δ .





- Indirect Reasoning
 - You are taking a multiple choice test.
 - You don't know the correct answer.
 - You eliminate the answers you know are incorrect.
 - The answer that is left is the correct answer.
- You can use the same type of logic to prove geometric things.

- Indirect Proof
 - Proving things by making an assumption and showing that the assumption leads to a contradiction.
 - Essentially it is proof by eliminating all the other possibilities.

5.6 Inequalities in Two Triangles and Indirect Proof Steps for writing indirect proofs Identify what you are trying to prove. Temporarily, assume the conclusion is false and that the opposite is true. Show that this leads to a contradiction of the hypothesis or some other fact. Point out that the assumption must be false, so the conclusion must be true.



Suppose you wanted to prove the statement "If x + y ≠ 14 and y = 5, then x ≠ 9." What temporary assumption could you make to prove the conclusion indirectly?

How does that assumption lead to a contradiction?

Assume x = 9

If x = 9, then x + y \neq 14. 9 + 5 \neq 14 \rightarrow 14 \neq 14. This is the contradiction

□ 338 #4-18 even, 22-34 even = 15 total

□ Extra Credit 341 #2, 4 = +2

5.6 Answers and Quiz

□ <u>5.6 Answers</u>

□ <u>5.6 Quiz</u>

